Pensieve header: A table of expanded MVA values for some small links. For a test of finite-typeness, see MultivariableAlexander-Testing.nb at AcademicPensieve/2009-03 and the bottom of this notebook.

In[1]:=
```
<< KnotTheory`
```

```
Loading KnotTheory` version of April 20, 2009, 14:18:34.482.
Read more at http://katlas.org/wiki/KnotTheory.
```

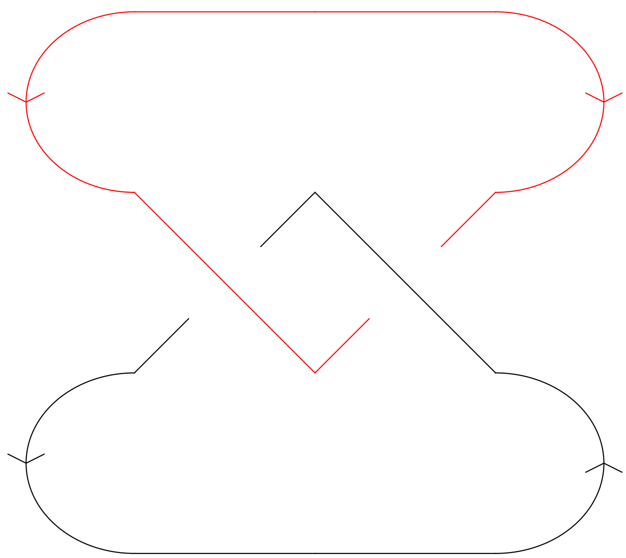
In[2]:=
```
Print[{
    #, DrawMorseLink[#],
    Expand[
     Normal[Series[
        MultivariableAlexander[#][t] /. t[i_] :> Exp[h x_i],
        {h, 0, 3}
       ]] /. h → 1
    ]
   }] & /@ AllLinks[{2, 8}];
```

KnotTheory::loading : Loading precomputed data in PD4Links`.

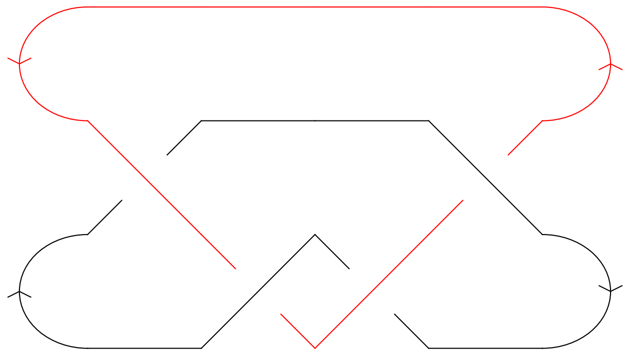
KnotTheory::credits : MorseLink was added to KnotTheory` by Siddarth Sankaran at the University of Toronto in the summer of 2005.

KnotTheory::credits : DrawMorseLink was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

KnotTheory::loading : Loading precomputed data in MultivariableAlexander4Links`.

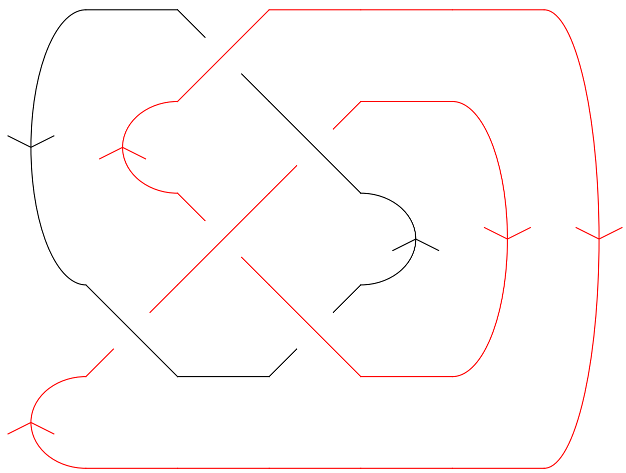
{Link[2, Alternating, 1],  , $-1 - x_1 - \frac{x_1^2}{2} - \frac{x_1^3}{6}$}

{Link[4, Alternating, 1],



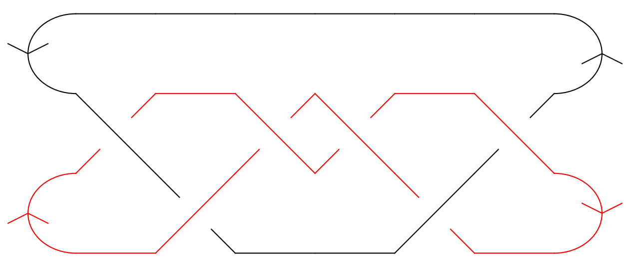
, $-2-\frac{x_1^2}{4}+\frac{x_1 x_2}{2}-\frac{x_2^2}{4}$}

{Link[5, Alternating, 1],



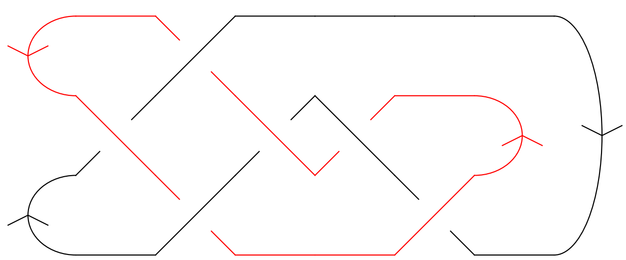
, $x_1 x_2$}

{Link[6, Alternating, 1],

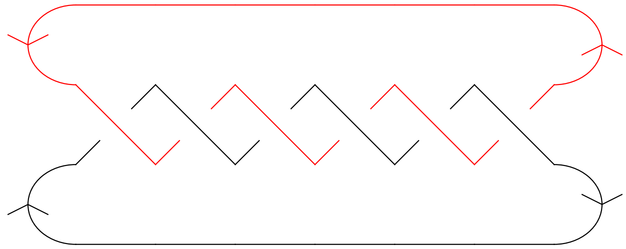


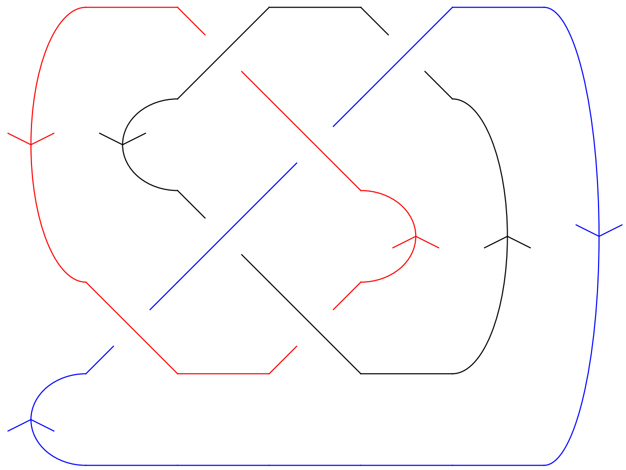
, $-2-\frac{x_1^2}{4}+\frac{3 x_1 x_2}{2}-\frac{x_2^2}{4}$}

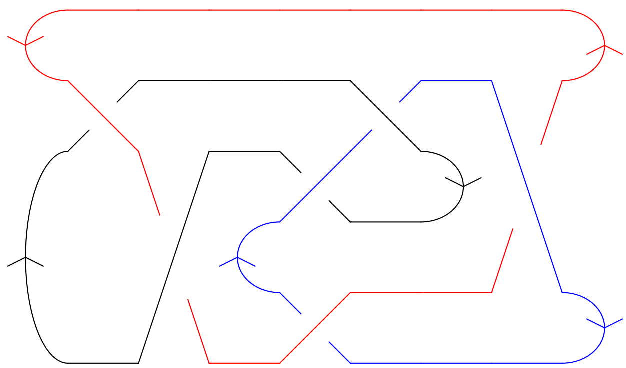
{Link[6, Alternating, 2],



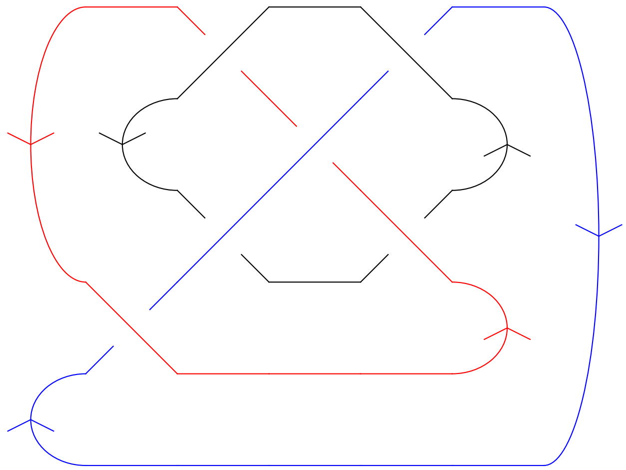
, $-3-x_1^2-x_2^2$}

{Link[6, Alternating, 3],  , $-3-x_1^2-2\,x_1\,x_2-x_2^2$}

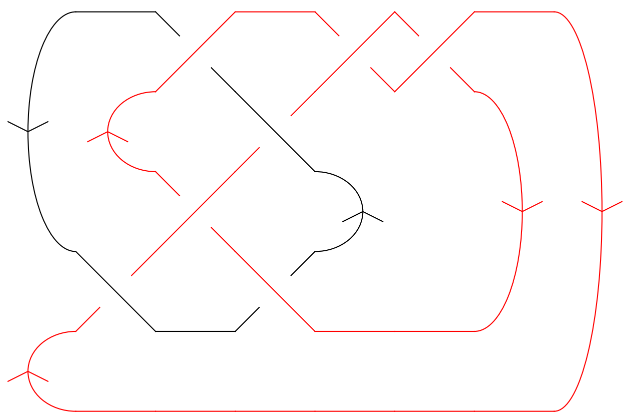
{Link[6, Alternating, 4],  , $x_1\,x_2\,x_3$}

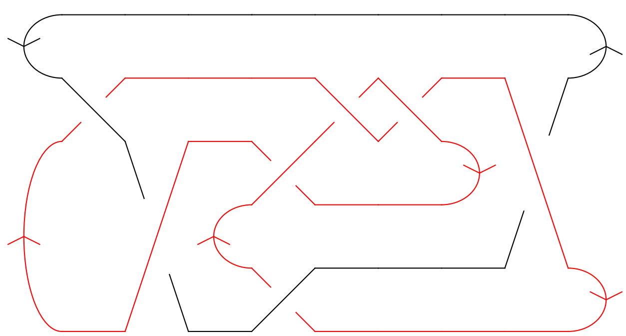
{Link[6, Alternating, 5],  ,

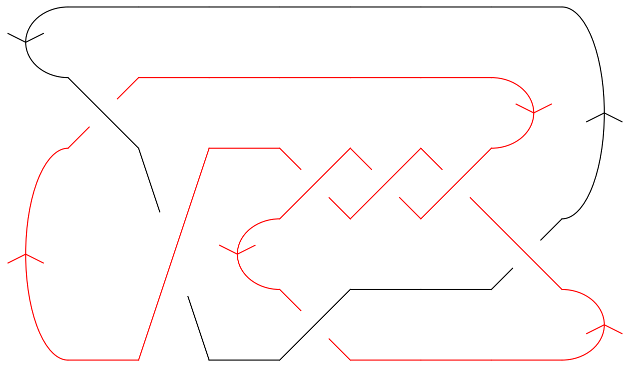
$x_1+\frac{x_1^3}{24}+x_2+\frac{1}{8}\,x_1^2\,x_2+\frac{1}{8}\,x_1\,x_2^2+\frac{x_2^3}{24}+x_3+\frac{1}{8}\,x_1^2\,x_3-\frac{3}{4}\,x_1\,x_2\,x_3+\frac{1}{8}\,x_2^2\,x_3+\frac{1}{8}\,x_1\,x_3^2+\frac{1}{8}\,x_2\,x_3^2+\frac{x_3^3}{24}$}

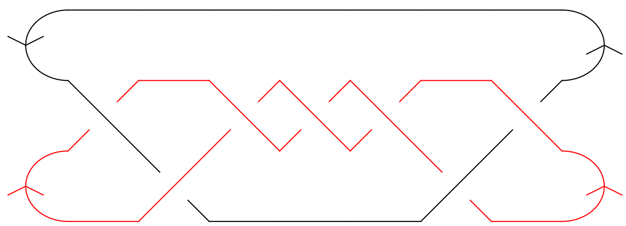
{Link[6, NonAlternating, 1],  ,

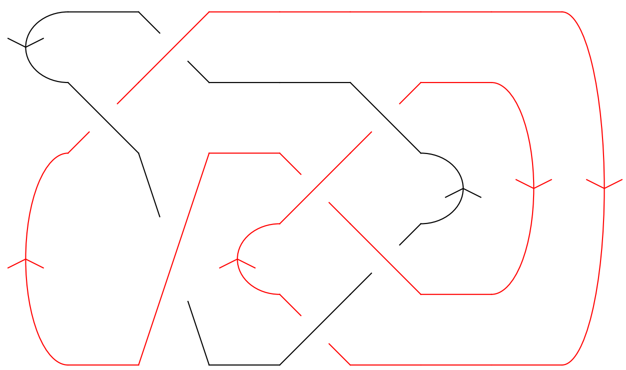
$-x_1-\frac{x_1^3}{24}-x_2-\frac{1}{8}x_1^2x_2-\frac{1}{8}x_1x_2^2-\frac{x_2^3}{24}+x_3+\frac{1}{8}x_1^2x_3+\frac{1}{4}x_1x_2x_3+\frac{1}{8}x_2^2x_3-\frac{1}{8}x_1x_3^2-\frac{1}{8}x_2x_3^2+\frac{x_3^3}{24}$}

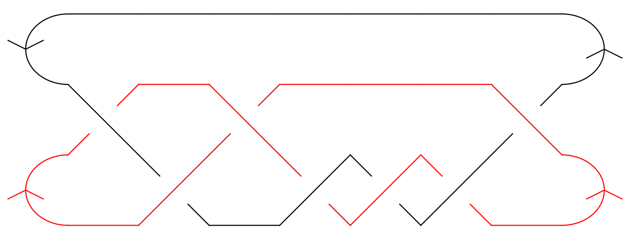
{Link[7, Alternating, 1],  , $x_1 x_2$}

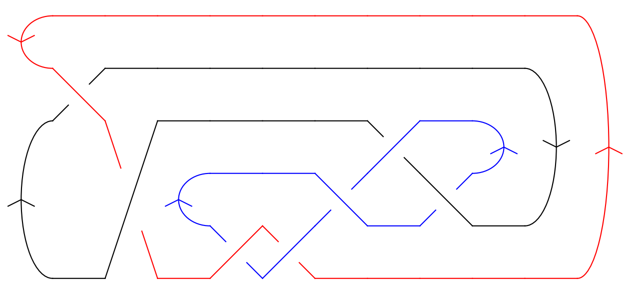
{Link[7, Alternating, 2],  , $-2-\frac{x_1^2}{4}-\frac{x_1 x_2}{2}-\frac{9 x_2^2}{4}$}

{Link[7, Alternating, 3],  , $-2 x_1 x_2$}

{Link[7, Alternating, 4],  , $-2 x_1 x_2$}

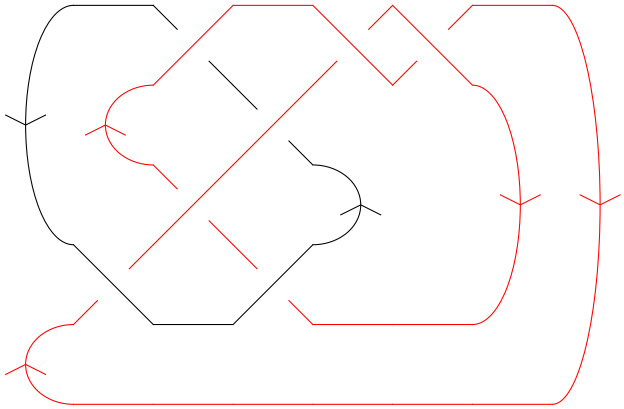
{Link[7, Alternating, 5],  , $-1 + 2 x_1 x_2$}

{Link[7, Alternating, 6],  , $1 - 2 x_1 x_2$}

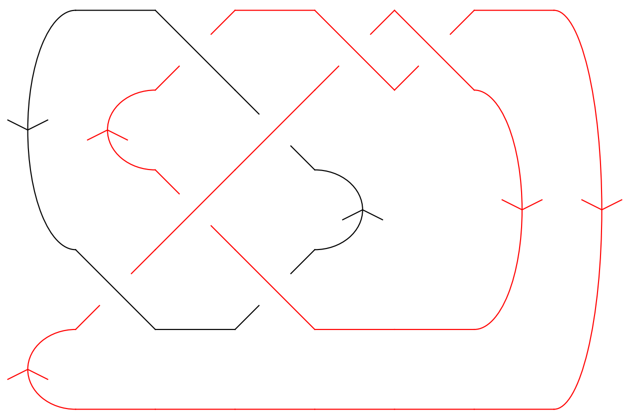
{Link[7, Alternating, 7],  ,

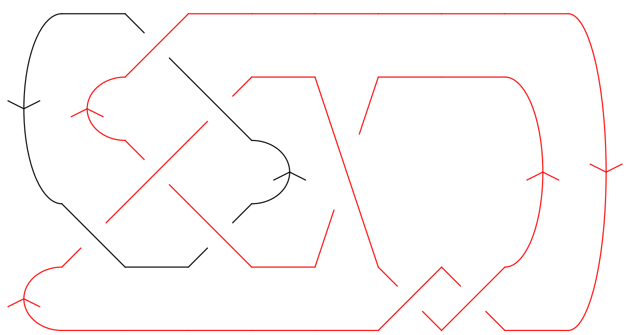
$x_1 + \frac{x_1^3}{24} - x_2 - \frac{1}{8} x_1^2 x_2 + \frac{1}{8} x_1 x_2^2 - \frac{x_2^3}{24} - x_3 - \frac{1}{8} x_1^2 x_3 + \frac{5}{4} x_1 x_2 x_3 - \frac{1}{8} x_2^2 x_3 + \frac{1}{8} x_1 x_3^2 - \frac{1}{8} x_2 x_3^2 - \frac{x_3^3}{24}$}

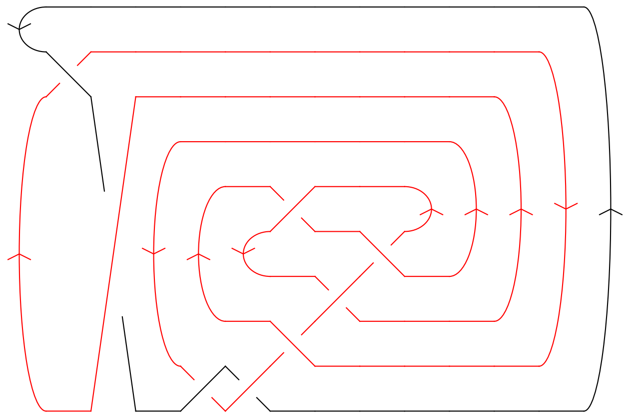
{Link[7, NonAlternating, 1],

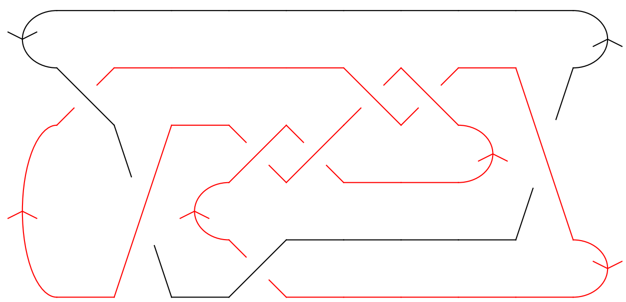


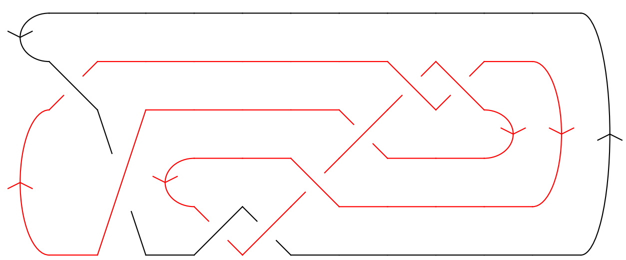
, $-2-\frac{x_1^2}{4}-\frac{3\,x_1\,x_2}{2}-\frac{9\,x_2^2}{4}$}

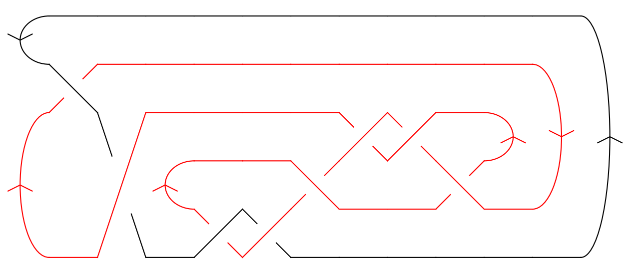
{Link[7, NonAlternating, 2],  , $x_1\,x_2$}

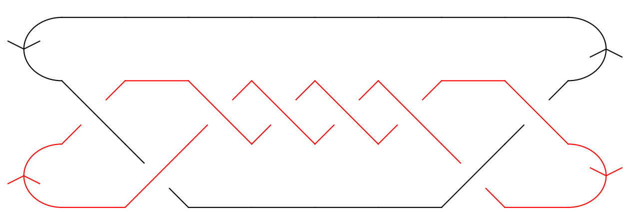
{Link[8, Alternating, 1],  , $x_1\,x_2$}

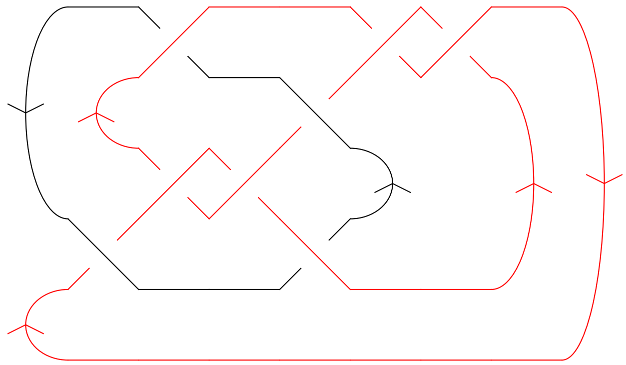
{Link[8, Alternating, 2],  , 0}

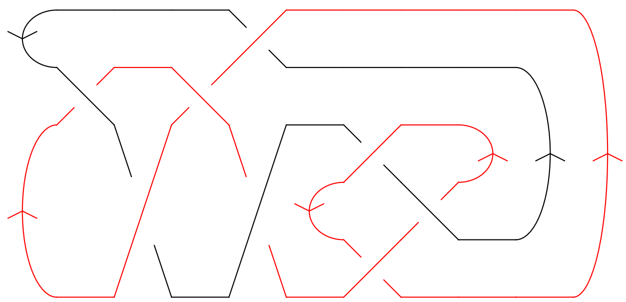
{Link[8, Alternating, 3],  , $-2-\frac{x_1^2}{4}+\frac{3\,x_1\,x_2}{2}+\frac{7\,x_2^2}{4}$}

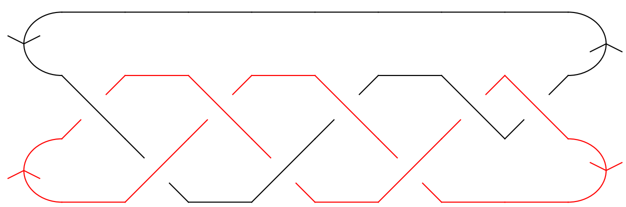
{Link[8, Alternating, 4],  , 0}

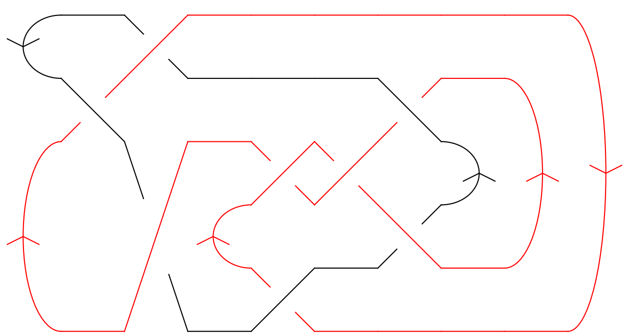
{Link[8, Alternating, 5],  , $-2-\frac{x_1^2}{4}+\frac{5\,x_1\,x_2}{2}-\frac{9\,x_2^2}{4}$}

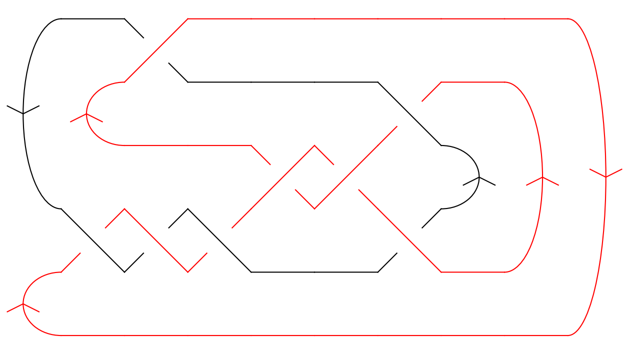
{Link[8, Alternating, 6],  , $-2-\frac{x_1^2}{4}+\frac{5\,x_1\,x_2}{2}-\frac{x_2^2}{4}$}

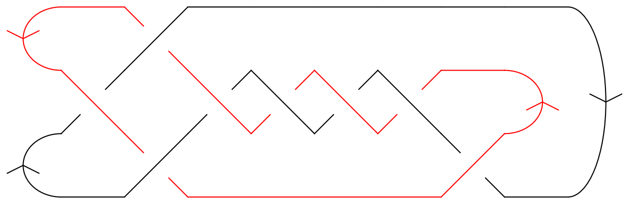
{Link[8, Alternating, 7],  , $-2 - \frac{x_1^2}{4} - \frac{5\, x_1\, x_2}{2} - \frac{9\, x_2^2}{4}$}

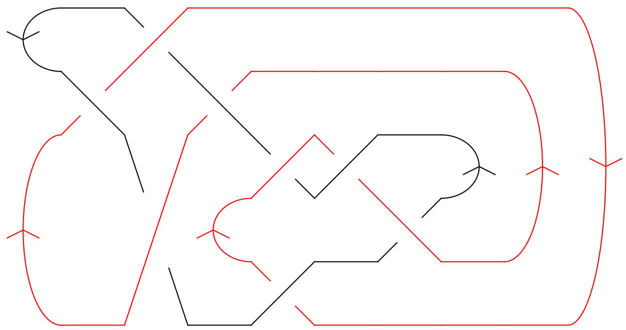
{Link[8, Alternating, 8],  , -1}

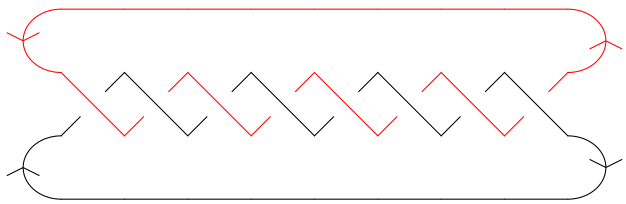
{Link[8, Alternating, 9],  , -1}

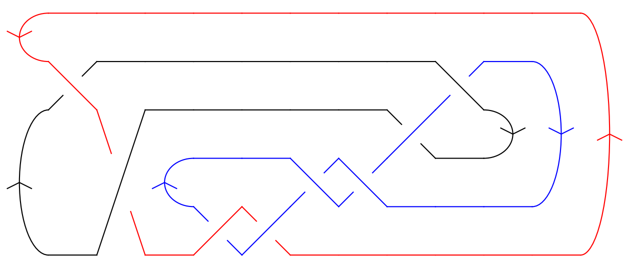
{Link[8, Alternating, 10],  , $-3 - x_1^2 - 2\, x_1\, x_2 - x_2^2$}

{Link[8, Alternating, 11],  , $-3 - x_1^2 - 4\, x_1\, x_2 - x_2^2$}

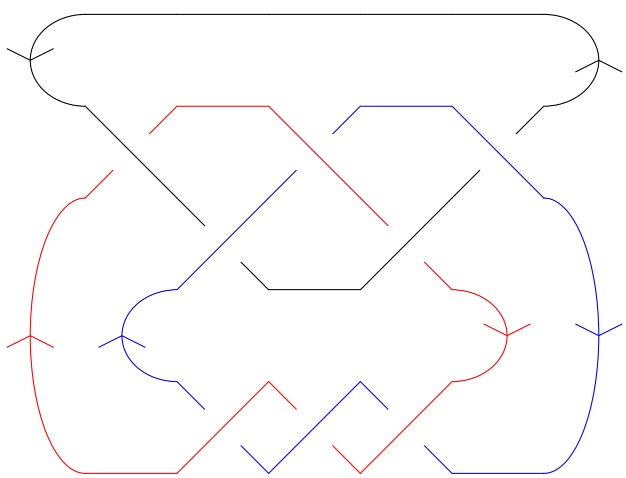
{Link[8, Alternating, 12],  , $-4 - \frac{5 x_1^2}{2} - 2 x_1 x_2 - \frac{5 x_2^2}{2}$}

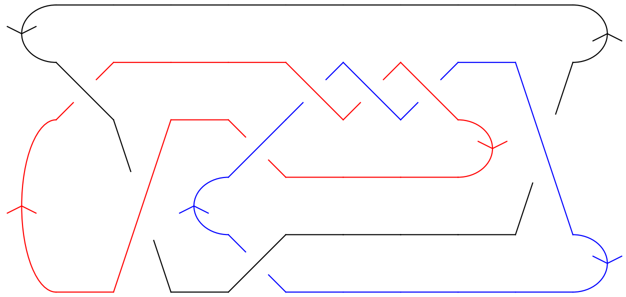
{Link[8, Alternating, 13],  , $-4 - \frac{5 x_1^2}{2} + x_1 x_2 - \frac{5 x_2^2}{2}$}

{Link[8, Alternating, 14],  , $-4 - \frac{5 x_1^2}{2} - 5 x_1 x_2 - \frac{5 x_2^2}{2}$}

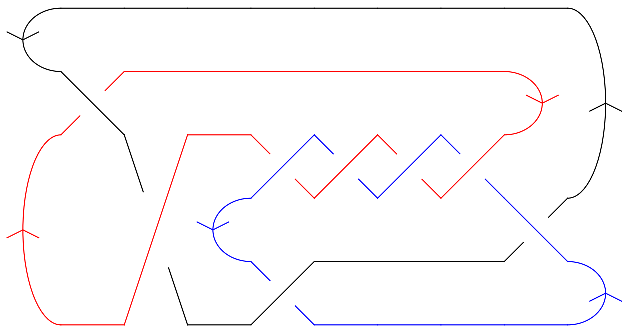
{Link[8, Alternating, 15],  ,

$x_1 + \frac{x_1^3}{24} + x_2 + \frac{1}{8} x_1^2 x_2 + \frac{1}{8} x_1 x_2^2 + \frac{x_2^3}{24} + x_3 + \frac{1}{8} x_1^2 x_3 - \frac{7}{4} x_1 x_2 x_3 + \frac{1}{8} x_2^2 x_3 + \frac{1}{8} x_1 x_3^2 + \frac{1}{8} x_2 x_3^2 + \frac{x_3^3}{24}$}

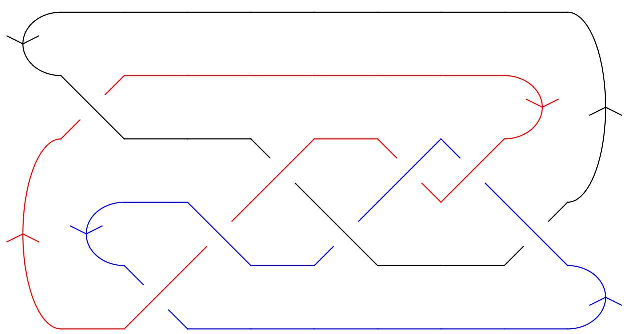
{Link[8, Alternating, 16],  , $2 x_1 x_2 x_3$}

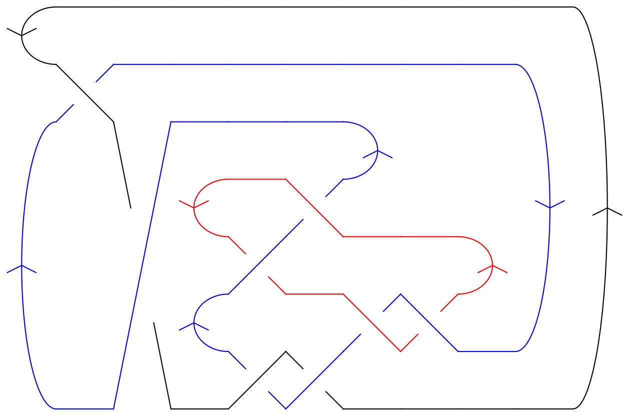
{Link[8, Alternating, 17], ,

$x_1+\frac{x_1^3}{24}+2x_2+\frac{1}{4}x_1^2x_2+\frac{1}{2}x_1x_2^2+\frac{x_2^3}{3}+2x_3+\frac{1}{4}x_1^2x_3-x_1x_2x_3+x_2^2x_3+\frac{1}{2}x_1x_3^2+x_2x_3^2+\frac{x_3^3}{3}$}

{Link[8, Alternating, 18], ,

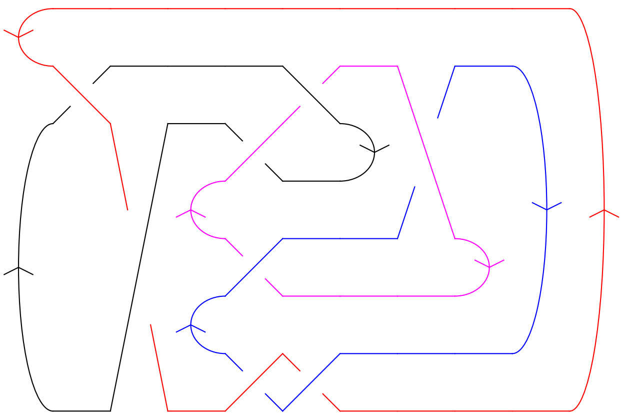
$-x_1-\frac{x_1^3}{24}-2x_2-\frac{1}{4}x_1^2x_2-\frac{1}{2}x_1x_2^2-\frac{x_2^3}{3}+2x_3+\frac{1}{4}x_1^2x_3-2x_1x_2x_3-\frac{1}{2}x_1x_3^2+\frac{x_3^3}{3}$}

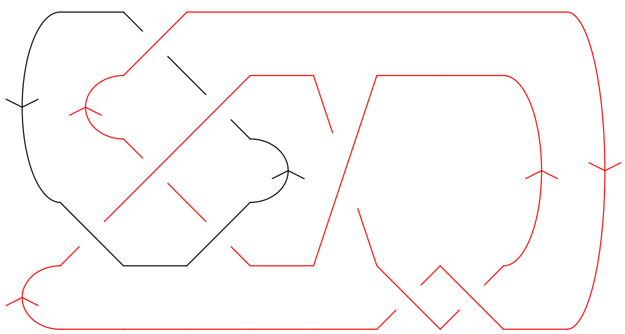
{Link[8, Alternating, 19], , $-x_1-\frac{x_1^3}{24}-2x_1x_2x_3$}

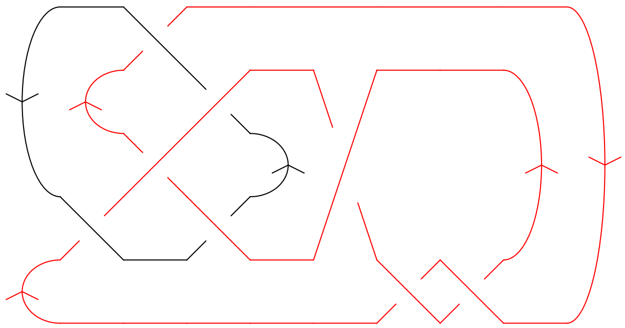
{Link[8, Alternating, 20],  ,

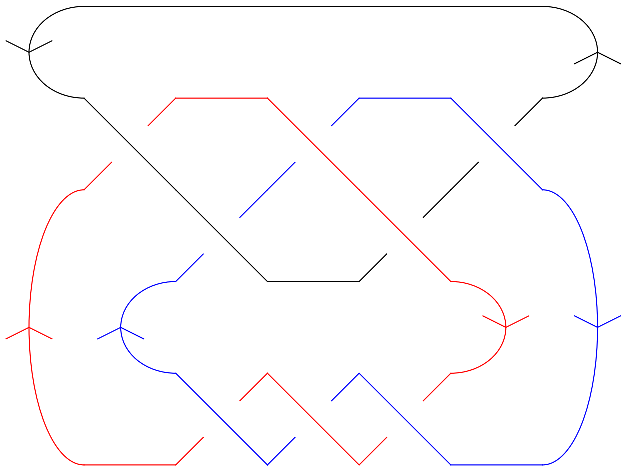
$-4\,x_3 - \frac{1}{2}\,x_1^2\,x_3 + x_1\,x_2\,x_3 - \frac{1}{2}\,x_2^2\,x_3 + x_1\,x_3^2 + x_2\,x_3^2 - \frac{7\,x_3^3}{6}$}

{Link[8, Alternating, 21],

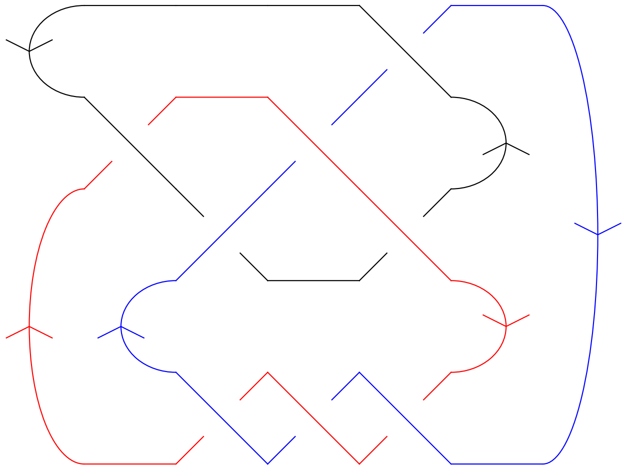
 , $-x_1\,x_2 - x_1\,x_3 - x_2\,x_4 - x_3\,x_4$}

{Link[8, NonAlternating, 1],  , $-2 - \frac{x_1^2}{4} + \frac{x_1\,x_2}{2} + \frac{7\,x_2^2}{4}$}

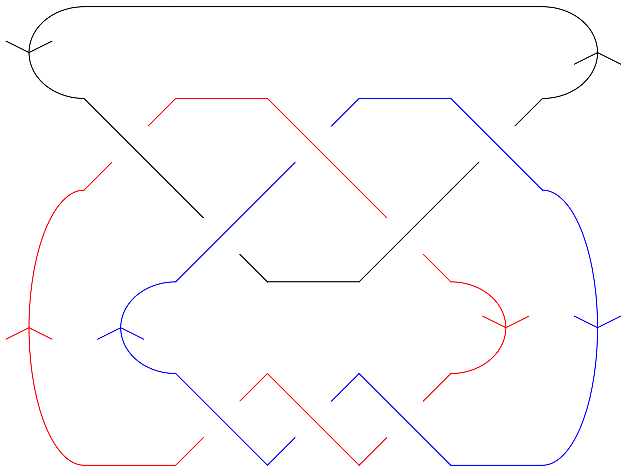
{Link[8, NonAlternating, 2],  , $-x_1 x_2$}

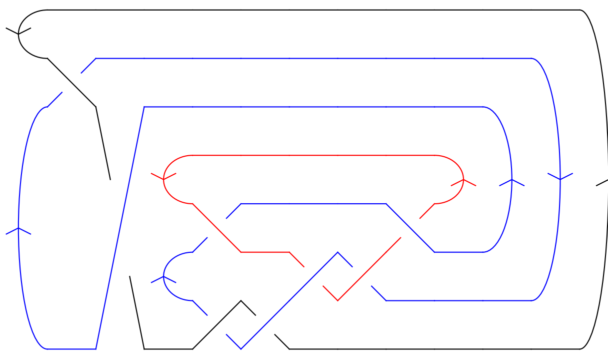
{Link[8, NonAlternating, 3],  ,

$$x_1 + \frac{x_1^3}{24} + 2 x_2 + \frac{1}{4} x_1^2 x_2 + \frac{1}{2} x_1 x_2^2 + \frac{x_2^3}{3} + 2 x_3 + \frac{1}{4} x_1^2 x_3 + x_1 x_2 x_3 + x_2^2 x_3 + \frac{1}{2} x_1 x_3^2 + x_2 x_3^2 + \frac{x_3^3}{3}$$ }

{Link[8, NonAlternating, 4],  ,

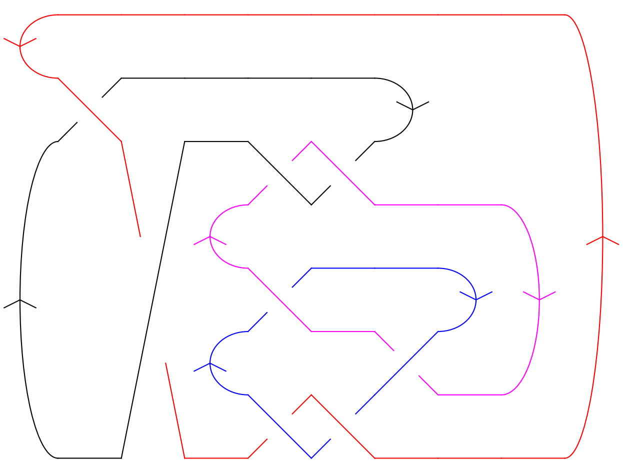
$$-x_1 - \frac{x_1^3}{24} + 2 x_2 + \frac{1}{4} x_1^2 x_2 - \frac{1}{2} x_1 x_2^2 + \frac{x_2^3}{3} - 2 x_3 - \frac{1}{4} x_1^2 x_3 - \frac{1}{2} x_1 x_3^2 - \frac{x_3^3}{3}$$ }

{Link[8, NonAlternating, 5],  , $-x_1 x_2 x_3$}

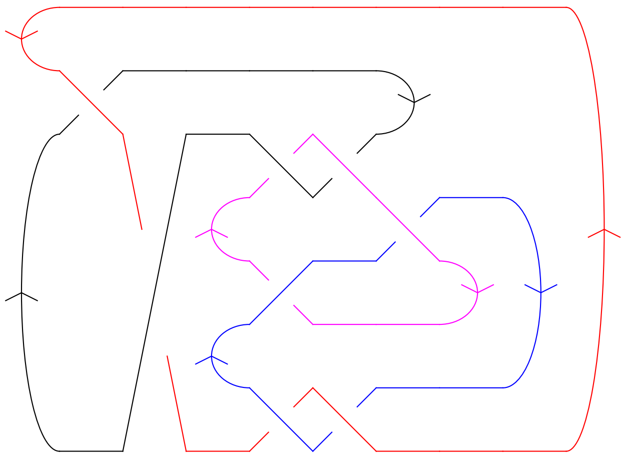
{Link[8, NonAlternating, 6],  ,

$4 x_3 + \frac{1}{2} x_1^2 x_3 + x_1 x_2 x_3 + \frac{1}{2} x_2^2 x_3 - x_1 x_3^2 - x_2 x_3^2 + \frac{7 x_3^3}{6}$}

{Link[8, NonAlternating, 7],

 , $-x_1 x_2 - x_1 x_3 - x_2 x_4 + x_3 x_4$}

{Link[8, NonAlternating, 8],



, $-x_1 x_2 + x_1 x_3 + x_2 x_4 - x_3 x_4$}

```
In[3]:= Flip[X[i_, j_, k_, l_]] := If[l == j + 1 || j - l > 1, X[j, k, l, i], X[l, i, j, k]];
        VCube[pd_, l_List] := Module[
           {f},
           Expand[pd * Times @@ ((1 - f[#]) & /@ l)] //. pd1_PD * f[i_] :> MapAt[Flip, pd1, i]
          ];
```

```
In[5]:= Print[# → Series[VCube[PD[#], {5}] /. pd_PD :> MultivariableAlexander[pd][t] /.
              t[i_] → E^(h x[i]), {h, 0, 0}]] & /@ AllLinks[8];
```

KnotTheory::credits :

The multivariable Alexander program "MVA2" was written by Jana Archibald at the University of Toronto in 2007–2008.

```
Link[8, Alternating, 1] → 1 + O[h]^1
Link[8, Alternating, 2] → O[h]^2
Link[8, Alternating, 3] → O[h]^2
Link[8, Alternating, 4] → O[h]^2
Link[8, Alternating, 5] → O[h]^2
Link[8, Alternating, 6] → O[h]^2
Link[8, Alternating, 7] → -1 + O[h]^1
Link[8, Alternating, 8] → O[h]^2
Link[8, Alternating, 9] → 1 + O[h]^1
Link[8, Alternating, 10] → -1 + O[h]^1
Link[8, Alternating, 11] → O[h]^2
Link[8, Alternating, 12] → -1 + O[h]^1
Link[8, Alternating, 13] → -1 + O[h]^1
Link[8, Alternating, 14] → -1 + O[h]^1
Link[8, Alternating, 15] → O[h]^2
Link[8, Alternating, 16] → O[h]^3
Link[8, Alternating, 17] → O[h]^1
Link[8, Alternating, 18] → O[h]^1
Link[8, Alternating, 19] → O[h]^1
Link[8, Alternating, 20] → O[h]^1
Link[8, Alternating, 21] → O[h]^2
Link[8, NonAlternating, 1] → -1 + O[h]^1
Link[8, NonAlternating, 2] → -1 + O[h]^1
Link[8, NonAlternating, 3] → O[h]^1
Link[8, NonAlternating, 4] → O[h]^1
Link[8, NonAlternating, 5] → O[h]^3
Link[8, NonAlternating, 6] → O[h]^1
Link[8, NonAlternating, 7] → O[h]^2
Link[8, NonAlternating, 8] → O[h]^2
```

```
In[6]:= Print[# → Series[VCube[PD[#], {4, 6}] /. pd_PD :> MultivariableAlexander[pd][t] /.
          t[i_] → E^(h x[i]), {h, 0, 1}]] & /@ AllLinks[8];
```

Link[8, Alternating, 1] → O[h]^2

Link[8, Alternating, 2] → O[h]^4

Link[8, Alternating, 3] → O[h]^2

Link[8, Alternating, 4] → O[h]^4

Link[8, Alternating, 5] → O[h]^2

Link[8, Alternating, 6] → O[h]^2

Link[8, Alternating, 7] → O[h]^2

Link[8, Alternating, 8] → O[h]^2

Link[8, Alternating, 9] → O[h]^2

Link[8, Alternating, 10] → O[h]^2

Link[8, Alternating, 11] → O[h]^2

Link[8, Alternating, 12] → O[h]^2

Link[8, Alternating, 13] → O[h]^2

Link[8, Alternating, 14] → O[h]^2

Link[8, Alternating, 15] → O[h]^2

Link[8, Alternating, 16] → x[3] h + O[h]^2

Link[8, Alternating, 17] → O[h]^2

Link[8, Alternating, 18] → O[h]^2

Link[8, Alternating, 19] → O[h]^2

Link[8, Alternating, 20] → O[h]^2

Link[8, Alternating, 21] → O[h]^2

Link[8, NonAlternating, 1] → O[h]^2

Link[8, NonAlternating, 2] → 0

Link[8, NonAlternating, 3] → x[3] h + O[h]^2

Link[8, NonAlternating, 4] → - x[3] h + O[h]^2

Link[8, NonAlternating, 5] → - x[3] h + O[h]^2

Link[8, NonAlternating, 6] → O[h]^2

Link[8, NonAlternating, 7] → O[h]^2

Link[8, NonAlternating, 8] → O[h]^2